Benha University
Faculty of Engineering- Shoubra
Electrical Engineering Department
2nd Year electrical Communication

Final Term Exam
Date: 12th of January 2012
Mathematics 3a
Duration : 3 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary.
- The exam. Consists of one page
- No. of questions:4
- Total Mark: 80 Marks

تنبيه: مراعاة إجابة كل جزء فى ناحية مستقلة

1-a)Find Fourier series for the function f(x) = | sin x | -π ≤ x ≤ π, and find $\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2(2m+1)^2}$.

1-b) Expand into complex Fourier series the periodic function f(x) = $\begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$ of period 2π

(15 marks)

2-a) Find generating function for Legendre polynomial and show that:

$$P_{n+1}(x) = \frac{2n+1}{n+1} x\, P_n(x) - \frac{n}{n-1} P_{n-1}(x), \text{ then find } P_2(x), P_3(x).$$

2-b) Write the reccurence relation for Bessel`s function and show that $\int x^n J_{n-1}(x)\, dx = x^n J_n(x) + c$, find $J_{5/2}(x)$ **(20 marks)**

Probability and Statistics (Total scores :45marks)

3a-i) Susan goes to work by one of two routes A or B. The prob. of going by route A is 30%. If she goes by route A, the prob. of being late is 5% and if she goes by route B, the prob. of being late is 10%. Given Susan is late for shool, find prob. that she went via route B. **(5 marks)**

3a-ii) Suppose that you have a bag filled with 50 marbles, 15 of which are green. What is the prob. of choosing exactly 3 green marbles if a total of 10 marbles are selected? **(5 marks)**

3b-i) Let the r.v. X be the distance in feet between bad records on a used computer tape. Suppose that a reasonable probability model for X is given by the p.d.f. f(x) = $\frac{1}{40} e^{-x/40}$, $0 < x < \infty$, find P(X > x) and then the median, also find m.g.f. and then deduce mean, standard deviation, μ_3, μ_3'. **(10 marks)**

3b-ii) Suppose that (X, Y) has probability density function f given by f(x, y) = (x + y) / 4 for 0 < x < y < 2

Find the probability density functions of X and of Y(Marginal of X and Y) , P[(X+Y) >1/2] and then determine if X and Y are independent. **(10 marks)**

4-a)A fair coin is tossed three times, X is the N^o of heads that come up on the first 2 tosses and Y is the N^o of heads that come up on tosses 2, 3. Construct the joint distribution and find marginal of X and Y, also find expected value and variance of X and Y **(10 marks)**

4-b) Suppose that if a car gets into an accident, the dollar amount of damage is Gamma-distributed with alpha = 2 and Beta = 100. Evaluate the average loss (in dollars) and the standard deviation of the loss (to the nearest dollar)? **(5 marks)**

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Model Answer

Probability and Statistics

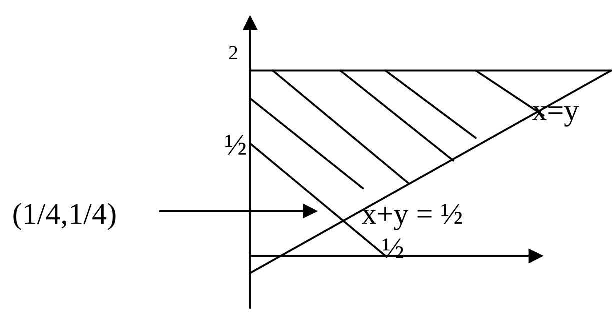
3a-i) P(A) = 0.3, P(B) = 0.7, P(L/A) = 0.05, P(L/B) = 0.1, P(B/L) = [P(L/B)P(B)]/P(L), where L: is Late event, P(L) = P(L/A)P(A) + P(L/B)P(B) = 0.05(0.3) + 0.1(0.7) = 0.085, so P(B/L) = 0.1(0.7)/0.085 = 0.824

3a-ii) Let the r.v. is the number of green marbles, so that by using hypergeometric distribution N = 50, k = 15, n = 10, therefore P(X = 3)= [${}^{50}C_3$] [${}^{35}C_7$] / [${}^{50}C_{10}$].

3b-i) P(X > x) = 1 – P(X<x) = $1-\int_0^x \frac{1}{40} e^{-x/40} dx = e^{-x/40}$ .To get the median a such that P(X< a) = 0.5, therefore $\int_0^a \frac{1}{40} e^{-x/40} dx = 0.5$, thus $1-e^{-a/40} = 0.5 \Rightarrow$ a = -40 ln(0.5) = 27.726, and m.g.f. = $\int_0^\infty e^{tx} (\frac{1}{40} e^{-x/40}) dx = \int_0^\infty \frac{1}{40} e^{\frac{-(1-40t)x}{40}} dx = \frac{1}{1-40t} = \phi(t)$, therefore $\mu_1' = \phi'(0) = \left.\frac{40}{(1-40t)^2}\right|_{t=0}$ = 40 = E(X) and $\mu_2' = \phi''(0) = \left.\frac{3200}{(1-40t)^3}\right|_{t=0}$ = 3200 = E(X^2), hence var(X) = E(X^2) – [E(X)]^2 = 3200 – 1600 = 1600, so standard deviation = 40 and $\mu_3' = \phi'''(0) = \left.\frac{384000}{(1-40t)^3}\right|_{t=0}$ = 384000, but $\mu_3 = \mu_3' - 3\mu_1' \mu_2' + 2[\mu_1']^3$, therefore μ_3 = 384000 -3 (40)(3200) + 2(40)^3 = 128000.

3b-ii) $f_1(x) = \int_x^2 \frac{(x+y)}{2} dy = \left.\frac{xy + y^2/2}{2}\right|_x^2 = x + 1 - \frac{3x^2}{4}$ and $f_2(y) = \int_0^y \frac{(x+y)}{2} dx = \left.\frac{xy + x^2/2}{2}\right|_0^y = \frac{3y^2}{4}$

P[(X+Y)>1/2] = $\int_{1/4}^{1/2} [\int_{1/2-y}^{y} (\frac{x+y}{2}) dx] dy + \int_{1/2}^{2} [\int_0^y (\frac{x+y}{2}) dx]dy$ and since $f_1(x) f_2(y) \neq f(x,y)$, thus they are not independent.



4-a)

Y \ X	0	1	2	$f_1(x)$
0	1/8	1/8	0	2/8
1	1/8	2/8	1/8	4/8
2	0	1/8	1/8	2/8
$f_2(y)$	2/8	4/8	2/8	1

E(X) = 0(2/8) + 1(4/8) + 2(2/8) = 1, E(Y) = 0(2/8) + 1(4/8) + 2(2/8) = 1, E(X^2) = 1(4/8) + 4(2/8) = 3/2 , E(Y^2) =1(4/8) + 4(2/8) = 3/2, Var(X) = Var(Y) = 1/2

4b) Since E(X) = α / β = 2/100 = 0.02, Var(X) = α / β^2, therefore standard deviation = $\sqrt{2}/100$